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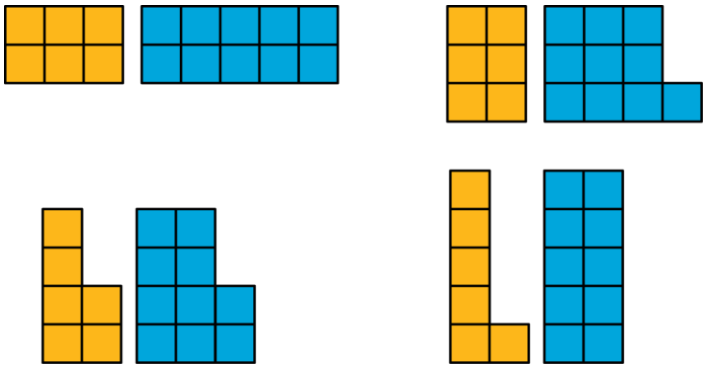
# Unit 7, Lesson 16: Common Factors

Let's use factors to solve problems.

## 16.1: Figures Made of Squares

How are the pairs of figures alike? How are they different?



## 16.2: Diego's Bake Sale

Diego is preparing brownies and cookies for a bake sale. He would like to make equal-size bags for selling all of the 48 brownies and 64 cookies that he has. Organize your answer to each question so that it can be followed by others.

1. How can Diego package all the 48 brownies so that each bag has the same number of them? How many bags can he make, and how many brownies will be in each bag? Find all the possible ways to package the brownies.

2. How can Diego package all the 64 cookies so that each bag has the same number of them? How many bags can he make, and how many cookies will be in each bag? Find all the possible ways to package the cookies.

3. How can Diego package all the 48 brownies and 64 cookies so that each bag has the same combination of items? How many bags can he make, and how many of each will be in each bag? Find all the possible ways to package both items.

4. What is the largest number of combination bags that Diego can make with no left over? Explain to your partner how you know that it is the largest possible number of bags.

## 16.3: Greatest Common Factor

1. The **greatest common factor** of 30 and 18 is 6. What do you think the term "greatest common factor" means?

2. Find all of the **factors** of 21 and 6. Then, identify the greatest common factor of 21 and 6.

3. Find all of the factors of 28 and 12. Then, identify the greatest common factor of 28 and 12.

4. A rectangular bulletin board is 12 inches tall and 27 inches wide. Elena plans to cover it with squares of colored paper that are all the same size. The paper squares come in different sizes; all of them have whole-number inches for their side lengths.

    a. What is the side length of the largest square that Elena could use to cover the bulletin board completely without gaps and overlaps? Explain or show your reasoning.

b. How is the solution to this problem related to greatest common factor?

### Are you ready for more?

A school has 1000 lockers, all lined up in a hallway. Each locker is closed. Then...

- One student goes down the hall and opens each locker.
- A second student goes down the hall and closes every second locker: lockers 2, 4, 6, and so on.
- A third student goes down the hall and changes every third locker. If a locker is open, he closes it. If a locker is closed, he opens it.
- A fourth student goes down the hall and changes every fourth locker.

This process continues up to the thousandth student! At the end of the process, which lockers will be open? (Hint: you may want to try this problem with a smaller number of lockers first.)

## Lesson 16 Summary

A factor of a whole number $n$ is a whole number that divides $n$ evenly without a remainder. For example, 1, 2, 3, 4, 6, and 12 are all factors of 12 because each of them divides 12 evenly and without a remainder.

A **common factor** of two whole numbers is a factor that they have in common. For example, 1, 3, 5, and 15 are factors of 45; they are also factors of 60. We call 1, 3, 5, and 15 common factors of 45 and 60.

The **greatest common factor** (sometimes written as GCF) of two whole numbers is the greatest of all of the common factors. For example, 15 is the greatest common factor for 45 and 60.

One way to find the greatest common factor of two whole numbers is to list all of the factors for each, and then look for the greatest factor they have in common. Let's try to find the greatest common factor of 18 and 24. First, we list all the factors of each number.

- Factors of 18: **1**, **2**, **3**, **6**, 9,18
- Factors of 24: **1**, **2**, **3**, 4, **6**, 8, 12, 24

The common factors are 1, 2, 3, and 6. Of these, 6 is the greatest one, so 6 is the greatest common factor of 18 and 24.

## Lesson 16 Glossary Terms

common factor greatest common factor

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# Unit 7, Lesson 17: Common Multiples

Let's use multiples to solve problems.

## 17.1: Notice and Wonder: Multiples

Circle all the multiples of 4 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Circle all the multiples of 6 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

What do you notice? What do you wonder?

## 17.2: The Florist's Order

A florist can order roses in bunches of 12 and lilies in bunches of 8. Last month she ordered the same number of roses and lilies.

1. If she ordered no more than 100 of each kind of flower, how many bunches of each could she have ordered? Find all the possible combinations.

2. What is the smallest number of bunches of roses that she could have ordered? What about the smallest number of bunches of lilies? Explain your reasoning.

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## 17.3: Least Common Multiple

The **least common multiple** of 6 and 8 is 24.

1. What do you think the term "least common multiple" means?

2. Find all of the **multiples** of 10 and 8 that are less than 100. Find the least common multiple of 10 and 8.

3. Find all of the multiples of 7 and 9 that are less than 100. Find the least common multiple of 7 and 9.

### Are you ready for more?

1. What is the least common multiple of 10 and 20?

2. What is the least common multiple of 4 and 24?

3. In the previous two questions, one number is a multiple of the other. What do you notice about their least common multiple? Do you think this will always happen when one number is a multiple of the other? Explain your reasoning.

## 17.4: Prizes on Grand Opening Day

Lin's uncle is opening a bakery. On the bakery's grand opening day, he plans to give away prizes to the first 50 customers that enter the shop. Every fifth customer will get a free bagel. Every ninth customer will get a free blueberry muffin. Every 12th customer will get a free slice of carrot cake.

1. Diego is waiting in line and is the 23rd customer. He thinks that he should get farther back in line in order to get a prize. Is he right? If so, how far back should he go to get at least one prize? Explain your reasoning.

2. Jada is the 36th customer.

   a. Will she get a prize? If so, what prize will she get?

   b. Is it possible for her to get more than one prize? How do you know? Explain your reasoning.

3. How many prizes total will Lin's uncle give away? Explain your reasoning.

## Lesson 17 Summary

A multiple of a whole number is a product of that number with another whole number. For example, 20 is a multiple of 4 because $20 = 5 \cdot 4$.

A **common multiple** for two whole numbers is a number that is a multiple of both numbers. For example, 20 is a multiple of 2 and a multiple of 5, so 20 is a common multiple of 2 and 5.

The **least common multiple** (sometimes written as LCM) of two whole numbers is the smallest multiple they have in common. For example, 30 is the least common multiple of 6 and 10.

One way to find the least common multiple of two numbers is to list multiples of each in order until we find the smallest multiple they have in common. Let's find the least common multiple for 4 and 10. First, we list some multiples of each number.

- Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, 32, 36, **40**, 44...
- Multiples of 10: 10, **20**, 30, **40**, 50, ...

20 and 40 are both common multiples of 4 and 10 (as are 60, 80, . . . ), but 20 is the smallest number that is on *both* lists, so 20 is the least common multiple.

## Lesson 17 Glossary Terms

least common multiple
common multiple

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# Unit 7, Lesson 18: Using Common Multiples and Common Factors

Let's use common factors and common multiple to solve problems.

## 18.1: Keeping a Steady Beat

Your teacher will give you instructions for playing a rhythm game. As you play the game, think about these questions:

- When will the two sounds happen at the same time?
- How does this game relate to common factors or common multiples?

## 18.2: Factors and Multiples

Work with your partner to solve the following problems.

1. **Party.** Elena is buying cups and plates for her party. Cups are sold in packs of 8 and plates are sold in packs of 6. She wants to have the same number of plates and cups.
   a. Find a number of plates and cups that meets her requirement.
   b. How many packs of each supply will she need to buy to get that number?
   c. Name two other quantities of plates and cups she could get to meet her requirement.

2. **Tiles**. A restaurant owner is replacing the restaurant's bathroom floor with square tiles. The tiles will be laid side-by-side to cover the entire bathroom with no gaps, and none of the tiles can be cut. The floor is a rectangle that measures 24 feet by 18 feet.
   a. What is the largest possible tile size she could use? Write the side length in feet. Explain how you know it's the largest possible tile.
   b. How many of these largest size tiles are needed?

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c. Name more tile sizes that are whole number of feet that she could use to cover the bathroom floor. Write the side lengths (in feet) of the square tiles.

3. **Stickers**. To celebrate the first day of spring, Lin is putting stickers on some of the 100 lockers along one side of her middle school's hallway. She puts a skateboard sticker on every 4th locker (starting with locker 4), and a kite sticker on every 5th locker (starting with locker 5).

   a. Name three lockers that will get both stickers.

   b. After Lin makes her way down the hall, will the 30th locker have no stickers, 1 sticker, or 2 stickers? Explain how you know.

4. **Kits.** The school nurse is assembling first-aid kits for the teachers. She has 75 bandages and 90 throat lozenges. All the kits must have the same number of each supply, and all supplies must be used.

   a. What is the largest number of kits the nurse can make?

   b. How many bandages and lozenges will be in each kit?

5. What kind of mathematical work was involved in each of the previous problems? Put a checkmark to show what the questions were about.

| problem | finding multiples | finding least common multiple | finding factors | finding greatest common factor |
|---|---|---|---|---|
| Party | | | | |
| Tiles | | | | |
| Stickers | | | | |
| Kits | | | | |

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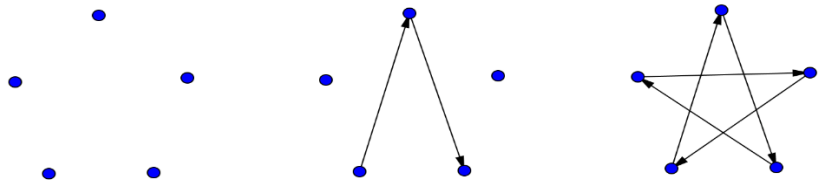
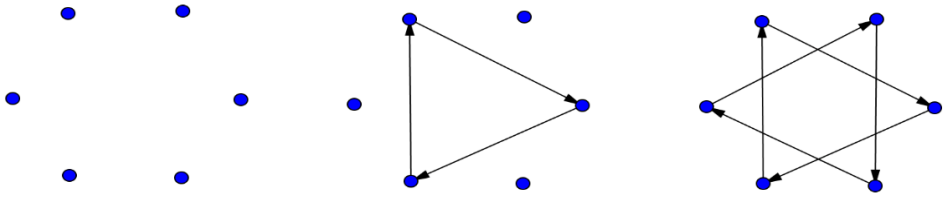
## Are you ready for more?

You probably know how to draw a five-pointed star without lifting your pencil. One way to do this is to start with five dots arranged in a circle, then connect every second dot.

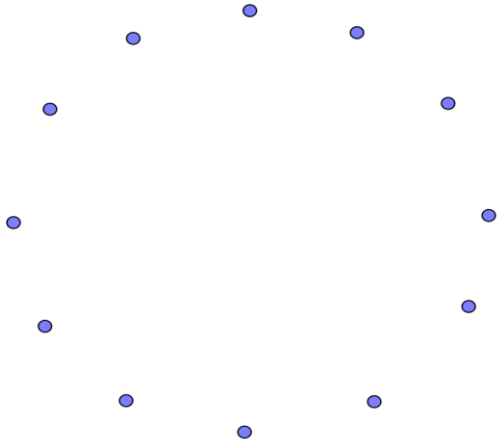


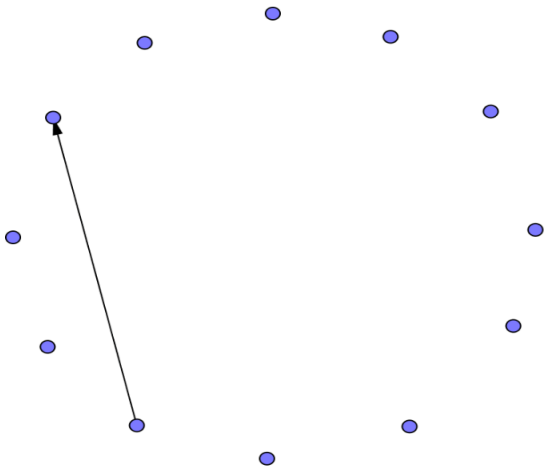
If you try the same thing with six dots arranged in a circle, you will have to lift your pencil. Once you make the first triangle, you'll have to find an empty dot and start the process over. Your six-pointed star has two pieces that are each drawn without lifting the pencil.



With twelve dots arranged in a circle, we can make some twelve-pointed stars.

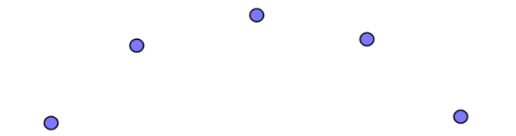
1. Start with one dot and connect every second dot, as if you were drawing a five-pointed star. Can you draw the twelve-pointed star without lifting your pencil? If not, how many pieces does the twelve-pointed star have?



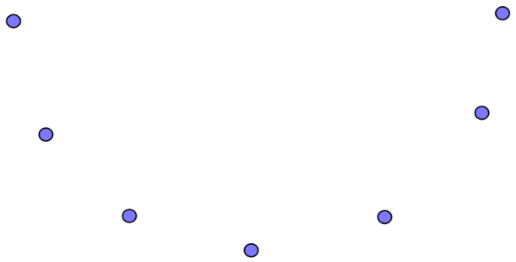


2. This time, connect every third dot. Can you draw this twelve-pointed star without lifting your pencil? If not, how many pieces do you get?

3. What do you think will happen if you connect every fourth dot? Try it. How many pieces do you get?



4. Do you think there is any way to draw a twelve-pointed star without lifting your pencil? Try it out.



5. Now investigate eight-pointed stars, nine-pointed stars, and ten-pointed stars. What patterns do you notice?

## 18.3: More Factors and Multiples

Here are five more problems. Read and discuss each one with your group. *Without solving*, predict whether each problem involves finding common multiples or finding common factors. Circle one or more options to show your prediction.

1. **Soccer.** Diego and Andre are both in a summer soccer league. During the month of August, Diego has a game every 3rd day, starting August 3rd, and Andre has a game every 4th day, starting August 4th.

   - common multiples
   - least common multiple
   - common factors
   - greatest common factor

   a. What is the first date that both boys will have a game?

   b. How many of their games fall on the same date?

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2. **Performances.** During a performing arts festival, students from elementary and middle schools will be grouped together for various performances. There are 32 elementary students and 40 middle-school students. The arts director wants identical groups for the performances, with students from both schools in each group. Each student can be a part of only one group.

   - common multiples
   - least common multiple
   - common factors
   - greatest common factor

   a. Name all possible groupings.

   b. What is the largest number of groups that can be formed? How many elementary school students and how many middle school students will be in each group?

3. **Lights.** There is a string of holiday lights with red, gold, and blue lights. The red lights are set to blink every 12 seconds, the gold lights are set to blink every 8 seconds, and the blue lights are set to blink every 6 seconds. The lights are on an automatic timer that starts each day at 7:00 p.m. and stops at midnight.

   - common multiples
   - least common multiple
   - common factors
   - greatest common factor

   a. After how much time with all 3 lights blink at the exact same time?

   b. How many times total will this happen in one day?

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4. **Banners.** Noah has two pieces of cloth. He is making square banners for students to hold during the opening day game. One piece of cloth is 72 inches wide. The other is 90 inches wide. He wants to use all the cloth, and each square banner must be of equal width and as wide as possible.

   - common multiples
   - least common multiple
   - common factors
   - greatest common factor

   a. How wide should he cut the banners?

   b. How many banners can he cut?

5. **Dancers.** At Elena's dance recital her performance begins with a line of 48 dancers that perform in the dark with a black light that illuminates white clothing. All 48 dancers enter the stage in a straight line. Every 3rd dancer wears a white headband, every 5th dancer wears a white belt, and every 9th dancer wears a set of white gloves.

   - common multiples
   - least common multiple
   - common factors
   - greatest common factor

   a. If Elena is the 30th dancer, what accessories will she wear?

   b. Will any of the dancers wear all 3 accessories? If so, which one(s)?

   c. How many of each accessory will the dance teacher need to order?

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## Lesson 18 Summary

If a problem requires dividing two whole numbers by the same whole number, solving it involves looking for a common factor. If it requires finding the *largest* number that can divide into the two whole numbers, we are looking for the *greatest common factor*.

Suppose we have 12 bagels and 18 muffins and want to make bags so each bag has the same combination of bagels and muffins. The common factors of 12 and 18 tell us possible number of bags that can be made.

The common factors of 18 are 1, 2, 3, and 6. For these numbers of bags, here are the number of bagels and muffins per bag.

- 1 bag: 12 bagels and 18 muffins
- 2 bags: 6 bagels and 9 muffins
- 3 bags: 4 bagels and 6 muffins
- 6 bags: 2 bagels and 3 muffins

We can see that the largest number of bags that can be made, 6, is the greatest common factor.

If a problem requires finding a number that is a multiple of two given numbers, solving it involves looking for a common multiple. If it requires finding the *first* instance the two numbers share a multiple, we are looking for the *least common multiple*.

Suppose forks are sold in boxes of 9 and spoons are sold in boxes of 15, and we want to buy an equal number of each. The multiples of 9 tell us how many forks we could buy, and the multiples of 15 tell us how many spoons we could buy, as shown here.

- Forks: 9, 18, 27, 36, 45, 54, 63, 72, 90. . .
- Spoons: 15, 30, 45, 60, 75, 90. . .

If we want as many forks as spoons, our options are 45, 90, 135, and so on, but the smallest number of utensils we could buy is 45, the least common multiple. This means buying 5 boxes of forks ($5 \cdot 9 = 45$) and 3 boxes of spoons ($3 \cdot 15 = 45$).